IEEE Guide for Radio Methods of Measuring Earth Conductivity

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Foreword

(This Foreword is not a part of IEEE Std 356-1974, Guide for Radio Methods of Measuring Earth Conductivity.)

This guide is the result of a survey of radio techniques employed for measuring the conductivity of the earth. Results are applicable to the design of communication channels that are influenced by the electrical characteristics of the earth. Emphasized are those techniques used at frequencies where the loss tangent is large.

The techniques are classed broadly into those which make use of drill holes into the earth and those which do not. Also, we distinguish between those methods that are passive and those that are active. In the latter case, the radio source is controlled.

Comments and critiques on the various techniques are included. The guide contains 68 references employed in the survey, plus appendixes which contain theoretical and practical details underlying some of the methods.

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Contents

IEEE Guide for Radio Methods of Measuring Earth Conductivity

1. Purpose and Scope

The earth's surface and the material below it affect both the point-to-point radio-wave propagation above and below the earth's surface and the impedance of the antennas employed. For this reason the radio engineer is concerned with the macroscopic electrical properties of the earth. The properties of concern here are the conductivity and dielectric constant (or permittivity). We are also concerned with the horizontal stratification that may exist. However, we do not consider the effects of surface roughness in any detail.

In what follows, the magnetic permeability of the earth is taken to be that of free space $\mu_o = 4\pi \times 10^{-7}$ H/m. Exceptions, of course, occur in localized regions containing magnetic material for which the magnetic permeability μ differs from μ_o (for example magnetite).

The techniques for determining the electrical constants of the earth can be broadly divided into two categories:

(1) those where drill-hole access into the earth does not exist

(2) those where drill holes can be employed

Where a drill hole can be used, the methods for single drill holes give localized values of the constants in the vicinity of the hole. Propagation between two holes can be used to deduce bulk values of the electrical constants of the earth between. To characterize a broad area of the earth's surface would require data from an impractically large number of costly holes.

When interest is in the earth constants over a large area, such as coverage for broadcast stations, or where drill holes do not exist, techniques are employed which may be characterized as those which use *surface probes*.

The material in this guide is drawn from numerous articles and papers; most of it is available in the open literature. Useful summary texts are those of Keller and Frischknecht, Ref [1] and Grant and West, Ref [2]. Theoretical background is found in the prolific writings of Wait, including his book, Ref [3]. These sources treat for the most part surface-probing techniques, although one chapter of Keller and Frischknecht, Ref [1] treats some well-logging methods. Additional material is reported below with pertinent references (Ref [4]).

In Section 2, we discuss techniques using surface probes and in Section 3 we discuss methods suitable for drill holes (including well logging) and, additionally, a few miscellaneous specialized cases. In what follows we exclude discussion of those arrangements which employ gamma, x-ray radiation, or similar sources of radiation.

SI units are employed throughout this guide.

2. Surface-Measurement Techniques (Probes)

The earth sometimes may be considered as a homogeneous (ideal) semi-infinite medium or, in practice, horizontally stratified. The loss tangent for such a conductive medium is

$$p = \tan \Psi = \sigma / \omega \epsilon_o \epsilon_r \qquad \text{(Eq 1)}$$

where

$\epsilon_o \approx 10^{-9}/36\pi$ F/m

ω = the angular frequency = $2\pi f$

f = the wave frequency

ϵ_r = the real relative dielectric constant of the earth (or permittivity relative to that for free space).

For most cases of interest, the loss tangent is large.

As we have indicated, radio waves penetrate into the earth. The wave field strength attenuates exponentially with depth, the field strength decreasing to e^{-1} at the skin depth δ. For highly conducting media

$$\delta = (2/\omega\mu_o \sigma)^{1/2} \qquad \text{(Eq 2)}$$

Actually most methods give an *apparent conductivity* σ_a since the theory is based on idealization of the model.

Surface measurements can be divided into two categories:

(1) broad area coverage

(2) local areas

In the broad area category, described below, ground-wave propagation is the usual method. Local area measurements are of several types, as discussed later.

2.1 Broad Area Coverage. The coverage diagram of a broadcast station is critically dependent upon soil constants. Characterizing coverage areas or states or a country such as the United States requires extensive measurements. The usual measurements are those of measuring the fields from a broadcast station at many points on the surface along radials from the station, or near the surface by airborne receivers.

2.1.1 ***Theoretical Calculations.*** The predicted variation of field strength with distance, frequency, soil constants, antenna types, and heights is due to the theoretical work of Norton, Ref [5] and amplified by others such as Wait, Refs [6], [3]. Theoretical curves of field strength versus distance at a given frequency are drawn for several pairs of values of σ and ϵ_r. The measured data are then compared with the theoretical curves. Generally, a range of paired values is obtained. Measurements repeated at other frequencies assist in isolating more correct values of ϵ_r and σ.

Recent theoretical calculations using digital computers have been given by Berry and Chrisman, Ref [7] and by Gerks, Ref [8], based on Bremmer's original equations. Propagation over a stratified earth has been treated theoretically by Wait, Refs [6], [3].

2.1.2 ***Large Area Maps (Estimated Constants).*** Estimates of ϵ_r and σ for various types of ground soil (and water) are given in Norton, Ref [5], CCIR, Ref [9], Jordan and Balmain, Ref [10], and FCC publications, Ref [11]. The FCC has published a map of conductivity for the United States that is useful for broadcasters and for those concerned with radio point-to-point communication. A similar map is available for Canada. There are also standard field-strength curves published by the FCC and the U. S. Army Signal Corps.

These field-strength curves have been prepared for a spherical homogeneous earth. The constants deduced from a measurement are apparent or equivalent values in an actual case.

2.1.3 ***Wave-Tilt Method.*** One feature of ground-wave propagation, the *wave tilt*, can be used to deduce earth constants. Assuming for the moment a plane homogeneous earth, the ground-wave fields have two orthogonal electric components; one is the vertical field strength E_v and the other is a radial or longitudinal horizontal component E_h. The wave tilt, Ref [3], is the ratio

$$E_h/E_v \approx n^{-1}\,(1-n^{-2})^{1/2} \approx \frac{1}{n} \qquad \text{(Eq 3)}$$

where n is the (complex) refractive index of the earth relative to air given by

$$n = (\epsilon'_r)^{1/2} = (\epsilon_r)^{1/2}\,(1-j\,p)^{1/2} \qquad \text{(Eq 4)}$$

where

ϵ'_r = the complex relative dielectric constant

p = loss tangent given by Eq 1

ϵ_r = the real relative dielectric constant

Since n is complex, the resultant electric vector lies in the vertical plane and its end describes an ellipse tilted forward in the direction of propagation. A measure of the axial ratio and the tilt of the ellipse results in a measurement of n. For large loss tangents

$$n \approx (\sigma/\omega\epsilon_o)^{1/2} \exp(-j\pi/4) \qquad \text{(Eq 5)}$$

from which σ can then be deduced.

In the high-frequency band and for typical values of σ, n is still large and the elliptical properties are difficult to measure accurately.

An example of hf wave-tilt measurements in determining the electrical parameters of stratified earth is that of Eliassen, Ref [12].

2.1.4 ***Attenuation Method—Layered Earth Models.*** Maley, Ref [13], proposed a method to yield ϵ_r and σ for a two-layered earth from measurements of attenuation versus distance of the electric field from a vertical electric dipole. The theory is based upon that of Wait, Ref [3]. The method is applicable for numerical distances from 0.1 to 10 units, for which values of both ϵ_r and σ result. The procedure for data analysis involves a least-squares process on the data from a large number of measurements. Localized irregularities would then not seriously affect results.

Stanley, Ref [14], published results of field strength versus distance measurements at Point Barrow. The results were matched to theoretical curves for a two-layer model to deduce the electrical constants of the two layers and thickness of the upper layer.

A related phenomenon in ground-wave propagation is that found for a mixed earth, for example, propagation across a land-sea boundary. Millington, Ref [15], evolved a quasi-analytical method for predicting the change in field strength, with results from simulated microwave model measurement. Pressey *et al*, Ref [16], have given quasi-empirical methods for variations of phase of importance to navigational aid devices such as Decca. Wait, Refs [17] and [3], and Wait and Walters, Ref [18], have given the theoretical basis for mixed path ground-wave in both amplitude and phase. The enhancement (or decrease) crossing a boundary depends upon conductivity contrast (for example, land to sea).

2.2 Local-Area Methods. Instead of distances of tens or hundreds of kilometers separating transmitting and receiving antennas, more localized measurements involve separations of the order of 1 km, depending upon the technique. Two major categories exist depending upon whether active or passive methods are used.

2.2.1 ***Passive Methods.*** Passive methods involve measurement of natural fields of the earth, for example, magneto-telluric or telluric current methods; currents flow in the earth caused by variations in the earth's magnetic field. Perturbations to the earth's field at frequencies about 1 Hz are attributed to sferics or lightning discharges and exclude man-made noise sources. (While not strictly *radio* methods, these are included here for completeness.) The theory of these methods may be found in the texts by Keller and Frischknecht, Ref [1], or Wait, Ref [3]. Other references include papers by Cagniard, Ref [19], Cantwell and Madden, Ref [20], Lahiri and Price, Ref [21], Wait, Ref [22], Yungul, Ref [23], and Vozoff, Ref [24].

2.2.1.1 *Magneto-Telluric (M-T) Method.* In magneto-telluric methods the sources of the fields to be observed are natural telluric currents. The horizontal electric intensity E_h and the orthogonal magnetic intensity H_h are observed. At the earth's surface the apparent complex surface impedance of the earth looking downward is

$$\zeta_a = E_h / H_h \qquad \text{(Eq 6)}$$

assuming plane waves and uniform stratification. The observations are made at frequencies below 1 Hz at which the loss tangents of the media are large. Then

$$\zeta_a = (j\,\omega\,\mu_o / \sigma_a)^{1/2} \qquad \text{(Eq 7)}$$

for nonmagnetic media, where σ_a is the apparent conductivity of the earth. If the earth is assumed to be horizontally stratified, values of σ_a in Eq 7 may be used to determine the characteristics of those horizontal layers, depending upon the model assumed. Thus, for two-layer models (for example, an overburden of finite thickness above a semi-infinite rock medium) the equations and curves of Cagniard, Ref [19], Wait, Ref [6], and Wait and Conda, Ref [25] may be used for interpretation. For three-layer models, the relations of Cagniard, Ref [19], and the curves of Yungul, Ref [23] may be employed. Vozoff, Ref [24], shows how a generalized inversion technique can yield the stratification model.

E_h is readily observed by using a horizontal wire, usually fed at the center, with ends grounded via electrodes. The wire arms may be arranged in various dipole geometries. The magnetic field is more difficult to observe accurately. The simplest method makes use of the magnetic balance arrangement, with its bar magnet. A recent development is an optically pumped magnetometer (Ref [1], p 239).

2.2.1.2 *Telluric-Current Method.* In the telluric-current method only the horizontal electric field is measured, avoiding the troublesome measurement of the minute magnetic-field variations. In this method, measurements are made at a base station and at a remote site. Accurate timing is required and

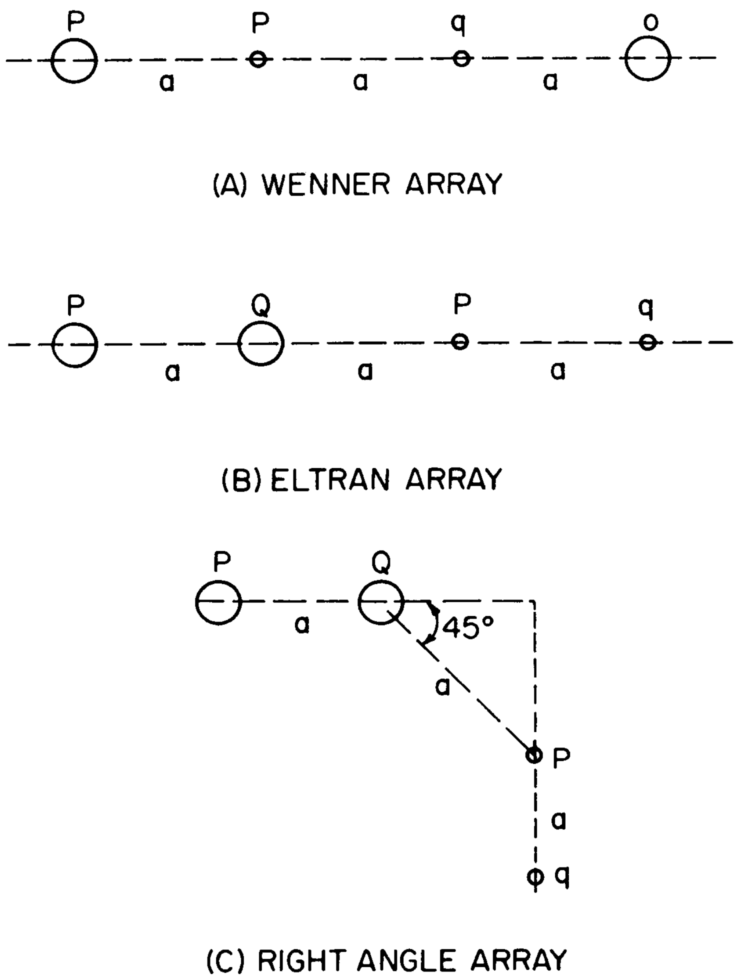


Fig 1
Four-Electrode Equispaced Arrays for Surface Resistivity Measurements. (A) Wenner Array, (B) Eltran Array, (C) Right-Angle or Wait Array. The Spacings are *a*. (From Wait and Conda, Ref [25])

measurements are usually those of micropulsations at frequencies less than 0.1 Hz. Further details may be found in Chapter 5 of Keller and Frischknecht, Ref [1].

These passive methods are used principally for measurements deep into the earth. To improve sensitivity special phase detectors have been developed (Ref [26]). Lahiri and Price, Ref [21], and more recently Cantwell and Madden, Ref [20], using M-T methods, found that at depths of the order of 100 km, the conductivity increased to values[1] between 0.03 and 1 S/m, presumably associated with the Mohorovičić Discontinuity *(Moho)*. Geophysicists prefer to use when possible active surface measuring methods rather than M-T methods.

2.2.2 *Active Methods.* In active methods of deducing earth constants from measurements at the earth's surface, the mutual impedance is measured between a transmitting antenna and a receiving antenna on the surface. In earlier days, a direct current was introduced

[1]The term "siemens" and the symbol "S" have been adopted for the unit of conductance previously referred to as the "mho." Use of this term has been approved by the IEEE, IEC, ISO, and Conference Generale des Poids et Mesures.

into one antenna, and the voltage detected in the second resulted in a value of mutual resistance which gave an apparent resistivity or conductivity of the earth. The antennas were simple wires.

With the advent of alternating currents, not only were wire antennas employed but loop arrangements were introduced. Various combinations of measuring E or H from loops or wires were usefully employed.

In addition to measurements with various arrays of loops or wire dipoles, another active method is the transmission line inserted into the earth. These methods are now discussed below.

2.2.2.1 *Four Electrode Arrangements: Wires.* Four electrode schemes with a variation discussed below are shown from the top schematically in Fig 1. Except for the right-angle array, these have been used since the 1920's (see Fritsch, Ref [27] and Schlumberger, Refs [28], [29]). Current is introduced into the earth through the *current* electrodes P and Q. The potential difference[2] resulting elsewhere is measured between *potential* electrodes p and q. In the Wenner array [See Fig 1(A)] P and Q are the outer electrodes, p and q the inner ones. In the Eltran arrangement [Fig 1(B)] the electrodes are also arranged on a line, but P and Q are at one end and p and q are at the other end. Fig 1(C) shows the modification to the Eltran arrangement introduced by Wait and Conda, Ref [25], and called the *right-angle* array. An advantage of the last is the absence of mutual inductance between *primary PQ* and *secondary pq*. The usual arrangements have all consecutive inter-electrode spacings equal, shown as a in the figure.[3] Computations of geometric factors for dc conductivity are based on potential theory (Refs [29], [30]).

As discussed by Wait and Conda, Ref [25], one measures the mutual resistance R_{12} from the measured primary current (I_1) and induced open-circuit voltage in the secondary (V_2). From a knowledge of electrode-circuit length and spacing between primary and secondary electrode circuits, one computes the apparent conductivity σ_a for the scheme used. The final geometric mean conductivity σ is obtained by curve matching to sets of master curves obtained from data as the spacing a is varied. The curves depend upon the model assumed and the type of electrode array employed. The curves are plots of $\sigma_a/\bar{\sigma}$ versus a/d_{eff} with reflection coefficient K as a parameter, where d_{eff} is the effective thickness of the upper layer of thickness d and α' is the transverse anisotropy σ_h/σ_v (see Fig 2). Curves for the two-layer model (Wenner array) were developed some time ago and recently were given for all three arrays in the above notation by Wait and Conda, Ref [25]. They show that the Eltran array is more sensitive at smaller spacings than the Wenner array but the latter is better at larger spacings. The right-angle array is somewhat in between the others for showing up sensitively departures of $\sigma_a/\bar{\sigma}$ with a/d_{eff}. Master curves have been computed for a three-layer model (see, for example, Hummel, Ref [31]). For the two-layer case, one computes σ_a from a knowledge of measured mutual resistance R_{12} and array spacing. When a is small, the measured R_{12} is due principally to the upper layer if its conductivity is larger than that of the lower region. If homogeneous regions are assumed and the upper layer thickness d is known, the curve of measured $\sigma_a/\bar{\sigma}$ versus a/d gives the value of K whence the value of σ_2 of the lower region.

Some preliminary measurements were made (Ref [32]) along a single line on Cape Cod. The technique employed the improved phase detector scheme aforementioned (Ref [26]. Insulated dipoles with special low resistance grounds were employed as the transmitting (current) and receiving (voltage) electrodes. The dipole lengths were about 300 m. Dipole centers were spaced from 610 to 2440 m in 300 m intervals.

Because of the anisotropic behavior alluded to earlier, certain care is needed in the analysis of the data. Strictly, for the survey of an area including the desired propagation path or an extensive site, one should make measurements over lines in several directions to ascertain horizontal anisotropy, especially in the elf region where the basement properties are important. Otherwise, single line results must be considered tentative. In all cases, it is

[2] The dc results depend only on electrode location while in active methods using the measurement of a distant field (see Section 2.2.2.4) the results depend on the position of the current-carrying wire.

[3] A modification of Wenner array is called the Lee array and is described in Chapter 3 of Keller and Frischknecht, Ref [1].

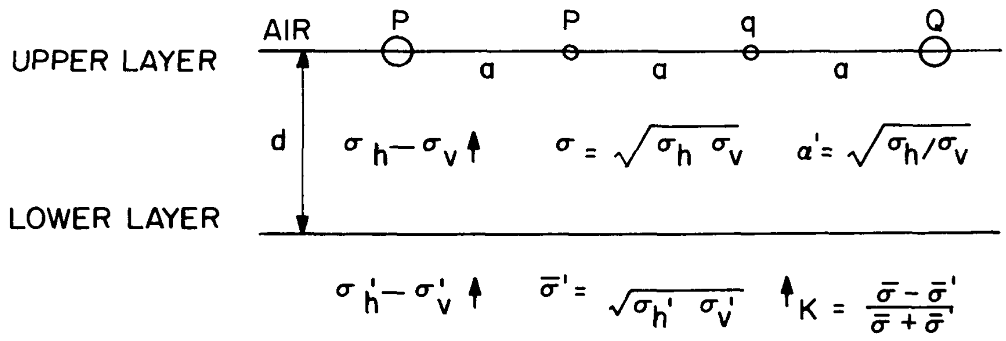


Fig 2
Schematic Representation for Two-Layer Ground, Each Layer Anisotropic. (Sketch Shows Wenner-Type 4-Electrode Arrangement)

necessary to select data for analysis to exclude local heterogeneities.

Note that in dc potential measurements, while the potential measured along the x axis of conductivity depends on $(\sigma_y \sigma_z)^{1/2}$, that along the y axis depends upon $(\sigma_x \sigma_z)^{1/2}$. In certain geological circumstances it is possible additionally to assume $\sigma_y \sim \sigma_z$ where $\sigma_x \leq \sigma_y$.

In the above measurements at 1 Hz it was deduced initially (Ref [32]) that the range of conductivity σ_1 of the upper layer, estimated from values for small spacings, was 7 to 10 mS/m for the overburden.[4] Comparing the graph of apparent resistivity versus electrode separation with master curves for a two-layer model (σ_2 is the rock conductivity), the graph agreed fairly well with master curves when the conductivity contrast $\sigma_2/\sigma_1 = 1/39$. One then deduces σ_1 to be 10 and σ_2 to be 2.6×10^{-4} S/m with layer thickness of about 150 m.

Marked distortion of the apparent conductivity contours was noted. The overburden was sand, clay, and gravel. Small-scale measurements to obtain a good estimate of overburden conductivity are apt to be quite variable. (Some of the variability noted near the drill holes may have been due to the casing pipe itself.)

Following the ideas of Wait and Conda, Ref [25], regarding anisotropy, the values of σ deduced above are those for an assumed homogeneous overburden and rock layers, that is, α' was assumed unity. The resulting value of σ should be corrected to its geometric mean value $\bar{\sigma}$ if α' were known.

Measurements should be made along other directions and better estimates of upper layer (overburden) conductivity are needed to obtain more final values of soil and rock constants.

The theory of the method of using long wires for making conductivity depth soundings is discussed by Wait and may be found in his text (Ref [3]) or with extensions described in Keller and Frischknecht, Ref [1]. Amplifications for practical use are given in the latter, with descriptions of apparatus and methods of analysis plus examples of experimental measurements. High voltages are required in the long dipole wires and thus precautions are needed for safety in the field. Low terminal electrode resistances are essential to keep voltage at safe levels.

Instead of arrays of horizontal electric dipoles, arrangements using loops may be employed to advantage, as discussed below.

2.2.2.2 *Loop Arrangements.* Excitation of an electrically small transmitting loop near the ground gives rise to several components of electric and magnetic fields at a distance along the ground. The recent theory is based on the work of Wait, Refs [33]-[35], who has extended the pioneering development of Sommerfeld and his formal integral solutions. A summary appears in Chapter 6 of Keller and Frischknecht, Ref [1]; see also Grant and

[4] See Footnote 1.

West, Ref [2]: What is measured is the mutual impedance Z_m between receiving antenna and transmitting loop; from this the apparent conductivity of the earth is deduced.

The value of Z_m depends upon the antenna configurations. A loop is referred to as *horizontal* if its plane lies in a horizontal plane parallel to the earth; or *vertical* if its plane lies in a plane normal to the earth.[5] A second loop as a receiver measures the magnetic field components of the transmitting loop. A wire dipole would be used to measure the electric field of the transmitting loop; for a horizontal transmitting loop, the wire would be horizontal and perpendicular to the direction of propagation.

Four loop-pair configurations are possible:

(1) horizontal coplanar
(2) vertical coplaner
(3) vertical coaxial
(4) perpendicular

as suggested by Wait, Ref [36], for depth soundings. The theory for horizontal coplanar loops or for a horizontal loop with electric wire dipole receiver is given in Appendix A, which discusses limitations.

Keller and Frischknecht, Ref [1], cite an example of measurements made at 400 Hz over thick ice floating in Arctic ocean water from which the conductivity of the sea water there was deduced as 3.2 S/m (see also Hohmann, Ref [37]). Difficulties of data analysis and the measurement over a range of frequencies are detailed in Williams and Benning, Ref [38], who published results of measurements by the University of New Mexico at elf, using horizontal and vertical loops and measuring two components of magnetic field. The theory was based on that of Wait, Ref [34], with the restrictions of the horizontal radial distance being much smaller than the free-space wavelength and larger than the skin depth. A further assumption was that the sum of transmitting and receiving loop heights was less than 0.07 skin depths; this implied heights of about 2 m or less. Let H_x be the magnitude of the magnetic field in the coaxial direction from a vertical loop and H_z the magnitude of the magnetic field in a vertical direction. Forming the ratio of measured fields as $Q = H_x/H_z$, then the conductivity is

$$\sigma = (C_1/f)\,(Q/R)^2 \qquad \text{(Eq 8)}$$

where

$C_1 = 285 \times 10^3$

R = the slant range between the receiver and the image in the ground of the transmitting loop

A value of 0.0115 S/m was deduced as the average of several measurements out to radial distances of 0.8 km. A cross check was obtained by forming a similar ratio Q for the horizontal loop, the constant being four times larger in an equation for conductivity similar to Eq 8. The method is said to have the advantages of compensation for linear calibration in the receiver and for variations in dipole moment of the source loop.

Another loop-pair method is known as the INPUT method of Barringer which is based on two specific suggestions of Wait, Refs [39], [40]. It is described in the text of Keller and Frischknecht, Ref [1], as well as those by Vanyan, Ref [41], and one edited by Wait, Ref [42]. INPUT is a system that examines the decay characteristics of the secondary voltage detected by a trailed loop due to a pulsed or half sine-wave excitation of the primary loop. The method is limited to location of massive ore zones or determining high conductivity contrasts in the earth. Generally, loop methods are limited to shallow depths because system sensitivity degrades rapidly (mutual impedance varies inversely with the cube of loop baseline distance).

2.2.2.3 *Transmission-Line Techniques.* Sections of parallel wire or coaxial transmission lines may be used for certain types of measurements of the electrical properties of dissipative media. When such lines are electrically short, they may be considered as parallel-wire or coaxial *condensers* (see King, Ref [43].)

An example of the use of parallel-wire lines is that reported by Kirkscether, Ref [44], for measuring the electrical properties of the ground. Iizuka and King, Ref [45], used a coaxial line filled with a dissipative liquid and measured the vhf properties of that liquid by slotted line and impedance techniques.

A coaxial condenser was also used. (Mea-

[5] The magnetic dipole moment is perpendicular to the plane of the loop; for a *horizontal* loop, such a loop is referred to by many authors as a *vertical magnetic dipole* (vmd). To avoid confusion, we shall use here the orientation of the plane of the loop, not its magnetic dipole moment *vector* direction, to describe loop arrangements.

surements of the properties of linear radiators immersed in such liquids were being studied and a knowledge of the constants of the liquid was required.) Von Hippel, Ref [46], has described such techniques.

The theory of the method is a straightforward application of transmission line equations, such as those given by King, Ref [47]. In Appendix B, application to measurements of the properties of well water in drill holes for studies in rock strata propagation is discussed (Ref [4]).

Kirkscether, Ref [44], measured ground-soil properties at 0.6 to 400 MHz using an open-circuited section of an unshielded balanced two-wire transmission line inserted into the ground. The wires were 3 mm in diameter spaced 22.2 mm center to center; the characteristic impedance in air was 323 Ω. Frequency variations of ϵ_r, conductivity, attenuation constant, and phase velocity were deduced and reported in curves. The frequency variations were quite large.

Transmission-line techniques, especially those using unterminated electrically short lines, have practical utility in field measurements. The media are relatively undisturbed by line insertion, the results being on the site data (Ref [4]). Applications are discussed also in Appendix B, including tests on vegetation (Ref [48]).

2.2.2.4 *H/I Method.* In this technique, often used at very low frequencies (quasi-static, near field), the absolute value of surface H field (H_y) at a distance d is measured with respect to the current I in a short horizontal wire of length s terminated via two end electrodes. When the condition of low frequency, low conductivity is met then it can be shown (Ref [49]) that the effective conductivity σ_{eff} under the wire referred to its geometrical position can be obtained from

$$H_y/I = (s/2\pi d^3)\,(j\omega\mu_o\,\sigma_{\text{eff}})^{-1/2}\cos\theta \qquad \text{(Eq 9)}$$

where θ is the observer's bearing with respect to the wire. The requirements are both that $d/\delta > 1$ and that ionospheric effects are not present, where δ is the skin depth calculated for σ_{eff}.

In practical field measurements, the wire length s should be at least twice the smallest skin depth anticipated.

Anisotropic effects on the fields measured by this and similar arrangements are discussed by Wait, Ref [50]. The effect of anisotropy on the azimuthal field pattern of a horizontal dipole has been calculated by Galejs, Ref [51].

3. Measurements in Drill Holes

In the previous section, methods were reviewed which are applicable to determining the electrical properties of the earth below by measurements using antennas on the surface, with sources being natural (for example, sferics) or antennas driven by transmitters. If drill holes exist, they may be used to advantage by methods to deduce ϵ_r and σ and their variation with depth. This is of interest in that radio propagation, especially for ground waves at medium and lower frequencies, is affected by horizontal stratification. The variations with depth can be deduced more directly from drill-hole data.

The methods for use in drill holes include:

(1) Bridge measurement of core samples
(2) Depth attenuation measurement
(3) Induction log
(4) Antenna impedance
(5) Path transmission loss

3.1 Core Samples. Core samples extracted from the ground during drilling of a bore hole may be cut into dielectric samples for measuring electrical constants on a radio frequency bridge.

Values of ϵ_r and loss tangent p (and hence σ) for dry-rock samples, for a wide range of frequency and temperature, have been available for some time. In recent years, emphasis has increased on laboratory efforts to measure electrical constants under conditions which hopefully approximate those encountered on the site.

For shallow rock beds, the principal on the site consideration is the water content, which can be simulated by soaking the rock samples in water (here one may refine the technique by soaking in water extracted from the drill holes).

An example of the variability with depth is that obtained from core samples of the Tubman Road (Cape Cod, Massachusetts) drill hole (Ref [4]). The core data were obtained by

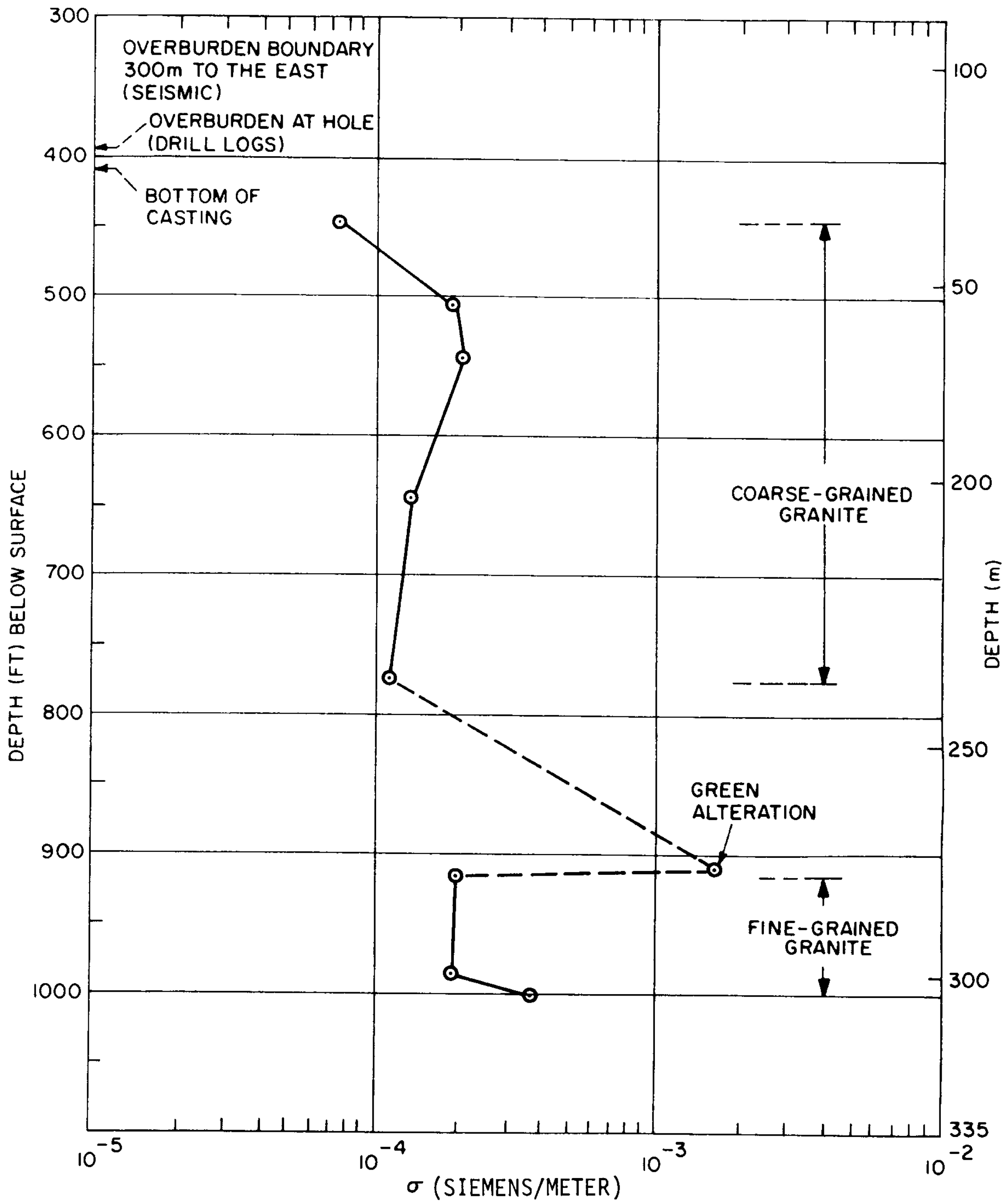


Fig 3
Conductivity of Core Samples Versus Depth-Tubman Road Drill Hole (Refs [4], [32])

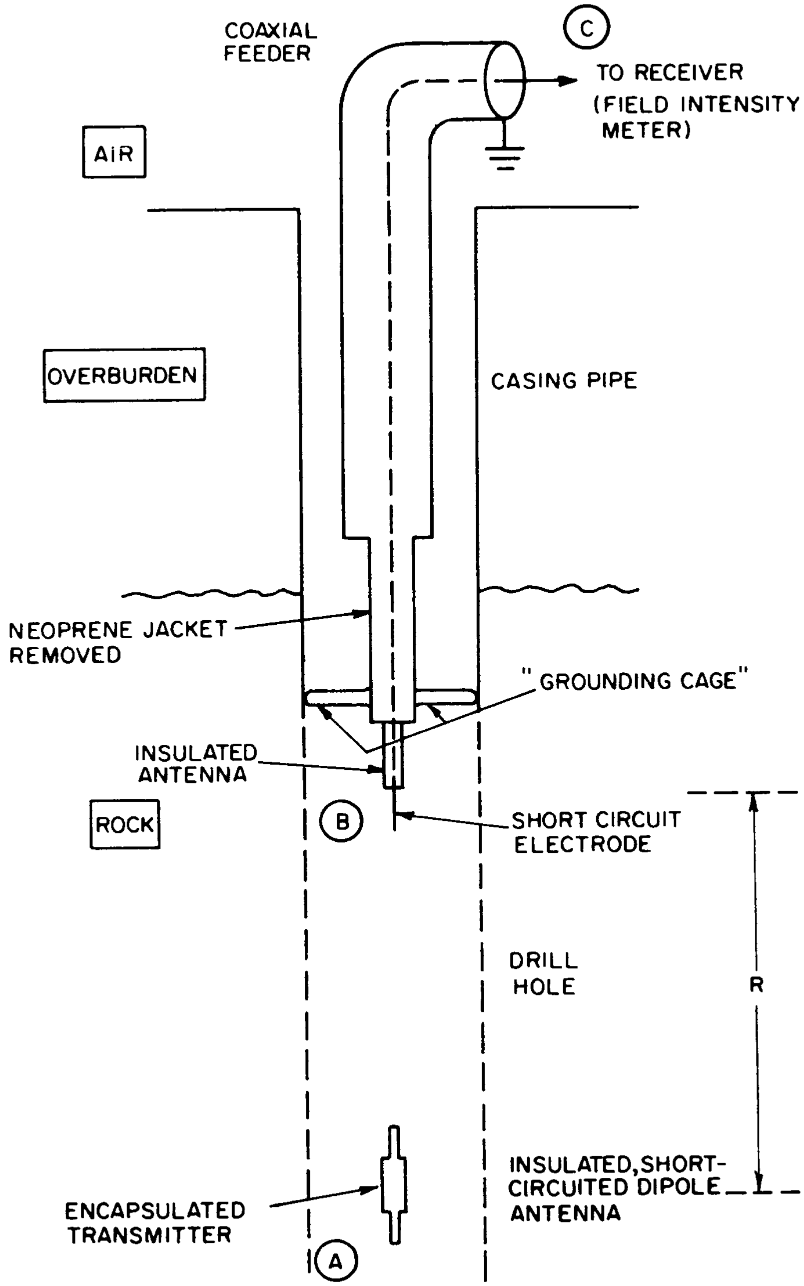


Fig 4
A Depth-Attenuation Method for Obtaining Electrical Constants of Rock Media. Transmitter at *A* Separated a Distance *R* From a Receiving Dipole at *B*, with *R* Varied. The Receiver is at *C* on Surface

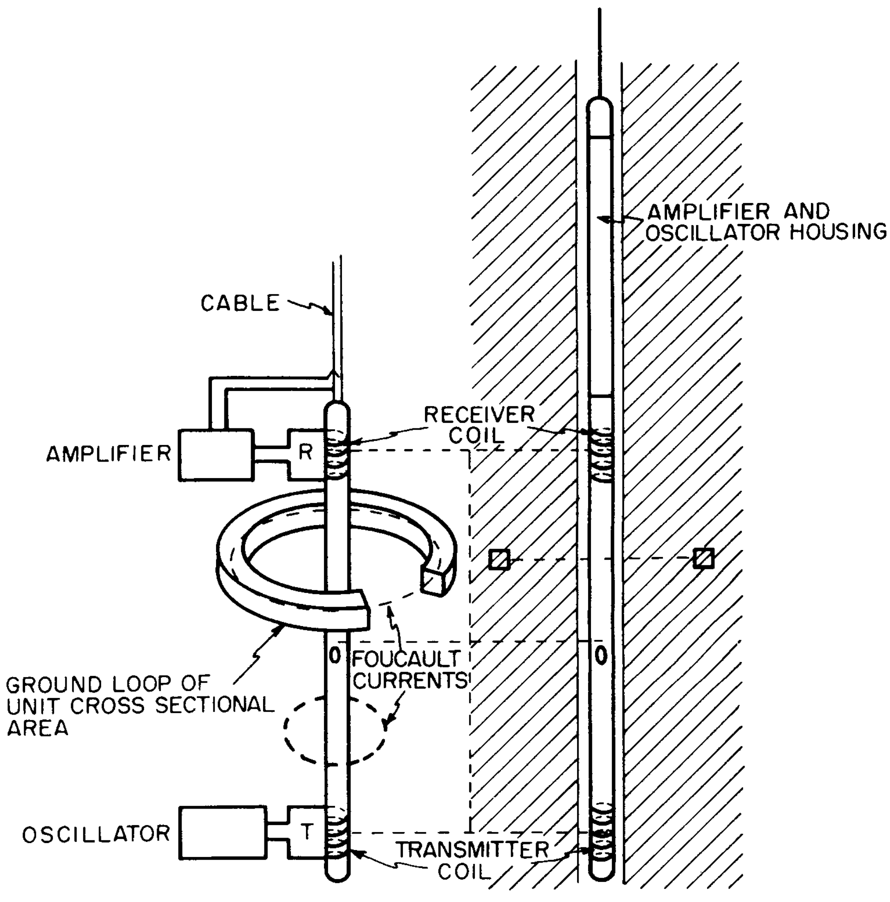
Cantwell and tabulated, (Ref [32]). The results shown in Fig 3 have an unweighted average conductivity of about 1.5×10^{-4} S/m which is in fair agreement with value of 2.6×10^{-4} S/m deduced from surface resistivity measurements.

3.2 Depth Attenuation Measurement. This technique is that of measuring the decay of the field in the rock media between separated transmitter and receiver as a function of the vertical distance of travel. One variant of the technique is illustrated (Ref [52]) in some detail in Fig 4. The depth attenuation method yields values of the attenuation constant.

The analysis is simplified if the distance between the transmitter and receiving antennas is greater than one wavelength (in the medium). This assumption limits the lowest frequency for useful observations in drill holes which have limited penetration depths into the rock.

3.3 Induction Logging. The induction log (Ref [53]) makes use of Foucault (eddy) current induced in the conducting medium by an

Fig 5
Principle of Operation of Induction Log (Left) and Arrangement of Components of Sonde (Right). (Ref [29])



alternating magnetic field. The induction log consists of a transmitting coil and a receiving coil and their associated electronic circuits. The coils are arranged to be coaxial along a cylindrical structure for lowering into the bore hole as a probe. The basic arrangement is shown in Fig 5. Due to the alternating current in the transmitting coil, Foucault current is induced in a circular strip of the conducting medium surrounding the induction log. This Foucault current in turn induces an emf in the receiving coil, the magnitude of which can then be related to the conductivity of the medium.

In a normal induction log, there will also be direct coupling between the transmitting and the receiving coil which may mask the effect of the Foucault current. This effect is minimized by a phase sensitive detector in the receiver. The detector is adjusted to produce no output when the induction log is in a nonconducting medium. Therefore, when the Foucault current effect is present, the receiver output would be a direct indication of conductivity.

The induction log is primarily useful for measuring high conductivity (Ref [28]), for example, $\sigma > 2 \times 10^{-2}$ S/m, although it is also useful with reduced accuracy for conductivity of 5×10^{-3} S/m or lower. One principal advantage of the induction log is its ability to function in an empty bore hole as well as in water-filled holes. It has found wide application in petroleum exploration.

In addition to the two-coil scheme shown in Fig 5, the induction log can be constructed with additional coils distributed in ways as to result in different geometrical form factor (array pattern) and, hence, the sensitivity of the response in overcoming effects of conducting mud columns which are generally present in drill holes in petroleum operation.

3.4 Antenna Input Impedance. In deep drill holes, linear wire radiators may be used to deduce the electrical properties of the medium from the measured antenna impedance. These radiators may be either electrically long or short.

Immersed insulated loops (Refs [54], [55]) have not been widely used as earth impedance probes because of the relative insensitivity of impedance to ground constants.

3.4.1 *Electrically Long Insulated Antenna.* Insulated wire antennas, with or without electrodes at the ends, have been a subject of many publications commencing perhaps with the work of Moore, Ref [56]. As long as the loss tangent of the medium is large, a useful approximation is to consider the immersed insulated wire as a coaxial line where the dissipative medium replaces the usual coaxial metallic outer conductor. A sinusoidal distribution of current is valid and the input impedance exhibits characteristics of a coaxial transmission line (Ref [4]). The quarter-wave resonance frequencies of these insulated antennas can be used to deduce the average conductivities of the rock surrounding the antennas (Refs [57], [4]).

3.4.2 *Electrically Short Conductivity Probe.* When the electric linear dipole is bare and electrically short, the input impedance yields local conductivity of the medium surrounding the antenna. Insulated wires terminated with electrodes may also be used (Refs [58]-[60]).

The impedance (and, hence, the conductivity) data of a short dipole indicate the presence or lack of stratification with a resolution commensurate with dipole dimension. Experimental difficulties include those associated with obtaining balanced center-fed arrangements with a feeder to the surface where the impedance measurements are made. Fig 6 shows the typical probe design used in a number of experiments reported in Refs [58], [61].

Generally, the conductivity probe is useful when there is conductive fluid, such as water, present in the drill hole to serve as contact between the dipole electrodes and the surrounding medium. The direct effect of water on the measurement results is mostly negligible if the thickness of the layer of water is small compared with the dipole length.

3.5 Path Transmission Loss Measurements. So far the methods described in this section are those which can be employed in single drill holes. Where several drill holes are available, a pair of these may be used in a transmission measurement to deduce the bulk conductivity of the medium between the drill holes. In this method, linear antennas or other appropriate form of radiators are inserted in the two drill holes for transmission and reception. The mutual impedance between the two antennas is the principal measured quantity for deducing

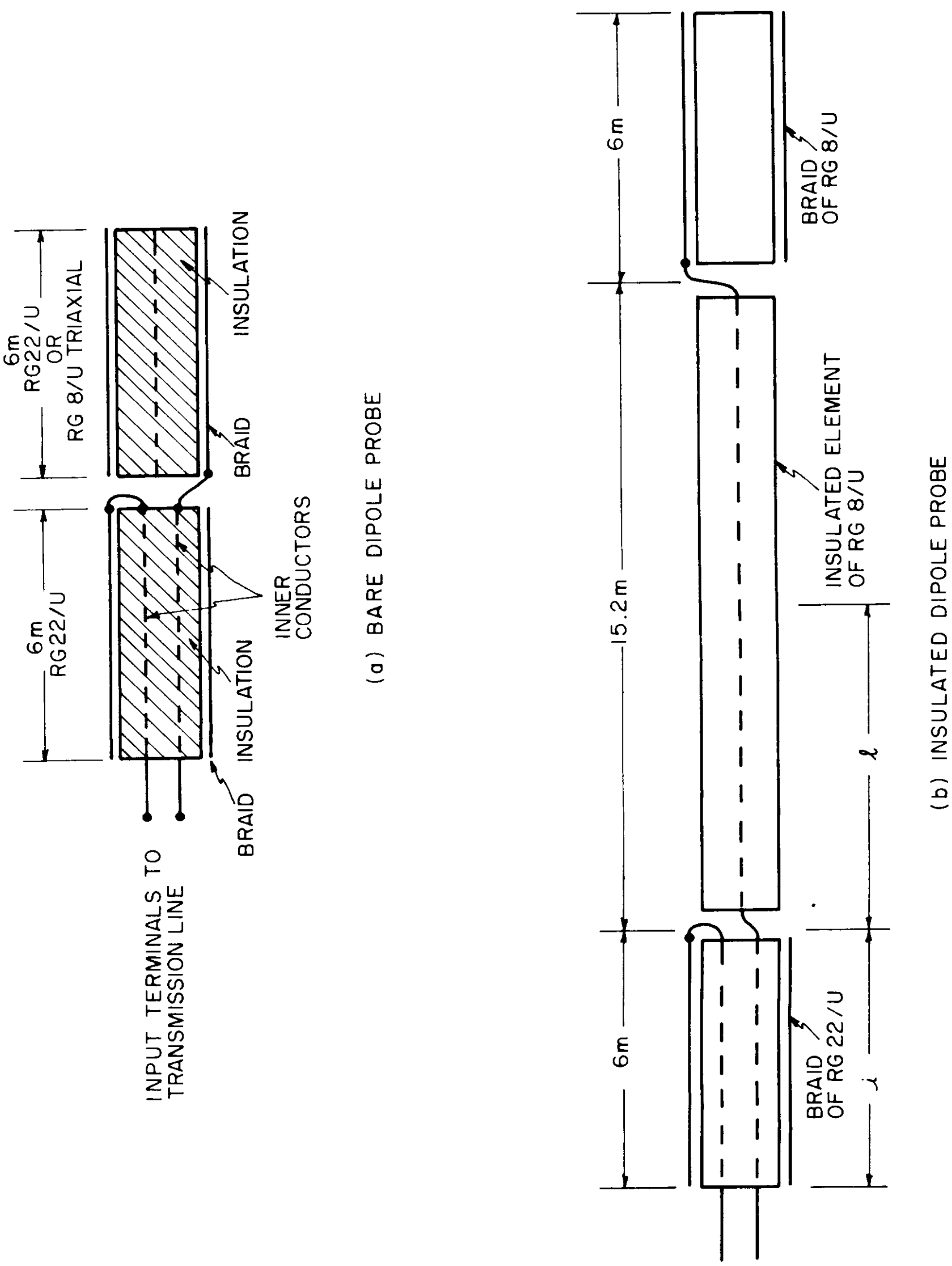


Fig 6
Electrically Short Linear Impedance Probes. (A) Bare Dipole Probe. (B) Insulated Dipole Probe.

the propagation loss and hence, the bulk electrical constants of the medium. This method is particularly useful if the measurement is made over a wide range of frequency.

The measurement of the mutual impedance can be made of the magnitude and/or the phase angle. Both have been applied successfully in a number of locations (Refs [57], [61], [63]). The magnitude of the mutual impedance is simpler to measure as only an incoherent instrumentation system is required; it can only result in attenuation constant and, therefore, the conductivity. If coherent instrumentation is used, the measurement of phase angle of the mutual impedance results in a phase constant which, for a medium of large loss tangent, yields also the conductivity, but for a medium with small loss tangent, gives the real relative dielectric constant.

The path transmission loss method (Refs [62]-[64]) is useful in areas where complex geology may render other techniques such as the surface resistivity method grossly ineffective in the evaluation of the underground electrical properties.

4. References[6]

[1] KELLER, G. V., and FRISCHKNECHT, F. C. *Electrical Methods in Geophysical Prospecting.* Oxford: Pergamon Press, 1966, chap IV – VII.

[2] GRANT, F. S., and WEST, G. F. *Interpretation Theory in Applied Geophysics.* New York: McGraw-Hill, 1965.

[3] WAIT, J. R. *Electromagnetic Waves in Stratified Media.* Oxford: Pergamon Press, 1962. 2nd enlarged edition 1970.

[4] deBETTENCOURT, J. T., and SUTCLIFFE, R. A. Studies in Deep Strata Communication. *Communications and Data Processing Operation,* The Raytheon Co, Norwood, Mass., Contract No AF19 (604)-8359; Oct 1962. (Final rprt for US Air Force Cambridge Research Lab, AD407 840.)

[5] NORTON, K. A. The Propagation of Radio Waves over the Earth and in the Atmosphere. pt I, *Proceedings of the IRE,* vol 24, Oct 1936, p 1367, and pt II, *Proceedings of the IRE,* vol 25, Sep 1937, p 1203.

[6] WAIT, J. R. Transmission and Reflection of Electromagnetic Waves in the Presence of Stratified Media. *NBS Journal of Research,* vol 61, Sep 1958, pp 205 – 232.

[7] BERRY, L. A., and CHRISMAN, M. E. A FORTRAN Program for Calculation of Groundwave Propagation over Homogeneous Spherical Earth for Dipole Antennas. *NBS Technical Report* 9178, 1966.

[8] GERKS, I. H. Use of a High-Speed Computer for Ground-Wave Calculation. *IRE Transactions on Antennas and Propagation,* vol AP-10, no 3, May 1962, pp 297 – 299.

[9] CCIR XII th Plenary Assembly, New Delhi, 1970. vol II, pt 1. *Propagation in Non-Ionized Media.* Rprt 229-1 (p 195) Determination of the Electrical Constants of the Surface of the Earth.

[10] JORDAN, E. C., and BALMAIN, K. G. *Electromagnetic Waves and Radiating Systems.* 2nd Ed. Englewood Cliffs, N.J.: Prentice-Hall, 1968, pp 632f.

[11] Federal Communications Commission. *Standards of Good Engineering Practice Concerning Standard Broadcast Stations, 550-1600 kc.* U. S. Gov't Printing Office, Wash. D. C., 1940. *Rules and Regulations,* vol III, Sep 1972, pt 73, Radio Broadcast Services.

[12] ELIASSEN, K. E. A Survey of Ground Conductivity and Dielectric Constant in Norway Within the Frequency Range (9.2 – 10 Mc/s). *Geofysiske Publikasjoner,* vol 19, no 11, 1957, pp 1 – 30.

[13] MALEY, S. W. A Method for the Measurement of the Parameters of a Two-Layer Stratified Earth. *IEEE Transactions on Antennas and Propagation,* vol AP-11, no 3, May 1963, pp 366 – 369.

[14] STANLEY, G. M. Layered Earth Propagation in the Vicinity of Point Barrow, Alaska. *NBS Journal of Research,* vol 64D, Jan 1960, pp 95 – 97.

[15] MILLINGTON, G. Ground-Wave Propagation Over an Inhomogeneous Smooth Earth. *Proceedings of the IEE* (London), vol 96, pt III, Jan 1949, pp 53 – 64.

[16] PRESSEY, B. G., ASHWELL, G. E., and FOWLER, C. S. Change of Phase of a Low-Frequency Ground Wave Propagated Across a Coast Line. *Proceedings of the IEE* (London) vol 103B, Jul 1956, p 527.

[6]AD numbered documents may be obtained from the National Technical Information Service (NTIS).

[17] WAIT, J. R. On the Theory of Mixed-Path Ground Wave Propagation on a Spherical Earth. *NBS Journal of Research,* vol 65D, no 4, Jul, Aug 1961, pp 401–410.

[18] WAIT, J. R., and WALTERS, L. C. Curves for Ground Wave Propagation Over Mixed Land and Sea Paths. *IEEE Transactions on Antennas and Propagation,* vol AP-11, no 1, Jun 1963, pp 38–45.

[19] CAGNIARD, L. Basic Theory of the Magneto–Telluric Method of Geophysical Prospecting. *Geophysics,* vol 18, Jul 1953, pp 605–635.

[20] CANTWELL, T., and MADDEN, T. R. Preliminary Report on Crystal Magneto–Telluric Measurements. *Journal of Geophysical Research,* vol 65, 1960, p 4202.

[21] LAHIRI, B. N. and PRICE, A. T. Electromagnetic Induction in Non-Uniform Conductors, and the Determination of the Conductivity of the Earth from Terrestrial Magnetic Variation. *Royal Society of London, Philosophical Transactions,* vol 237A, no 784, Jan 20, 1939, pp 509–540.

[22] WAIT, J. R. Theory of Magneto–Telluric Fields. *NBS Journal of Research,* vol 66D, Sep/Oct 1962, pp 509 – 541.

[23] YUNGUL, S. H. Magneto–Telluric Sounding Three-Layer Interpretation Curves. *Geophysics,* vol 26, Aug 1961, pp 465–476.

[24] VOZOFF, K. The Magnetotelluric Method in the Exploration of Sedimentary Basins. *Geophysics,* vol 37, no 1, Feb 1972, pp 98–141.

[25] WAIT, J. R., and CONDA, A. M. On the Measurement of Ground Conductivity at VLF. *IRE Transactions on Antennas and Propagation,* vol AP-6, Jul 1958, pp 273–277.

[26] MADDEN, T. R., CANTWELL, T., GREENEWELT, D., KELLY, A., and REGIER, A. Progress Report on Geomagnetic Studies and Electrical Conductivity in the Earth's Crust and Upper Mantle. MIT Geophysics Lab, Cambridge, Mass., Project Rprt NR-371-401, Apr 15, 1962.

[27] FRITSCH, V. *Elektrische Messungen an Räumlich Ausgedehnten Leitern.* Karlsruhe: G. Braun, 1960.

[28] Schlumberger Well Surveying Corp, *Introduction to Schlumberger Well Logging.* Houston, Texas, 4th printing, 1962.

[29] STEFANESCU, S., and SCHLUMBERGER, R. M. Electric Potential Distribution Around a Point in the Earth, *Journal de Physique et Radiation,* vol 1, 1930.

[30] SUNDE, E. D. *Earth Conduction Effects in Transmission Systems.* Princeton, N. J.: VanNostrand, 1949.

[31] HUMMEL, J. N. Apparent Resistivity in Surface Potential Methods. *AIME Geophysical Prospecting,* 1932, p. 392, also *Zeitschrift für Geophysik,* vol 5, 1929, p. 89, and vol 8, 1932, p 249.

[32] CANTWELL, T., and NELSON, P. Resistivity Investigation on Cape Cod. Geoscience, Inc., Cambridge, Mass., USAFCRL Rprt AFCRL-63-370, Scientific Rprt no 3-2351, Contract AF 19(628)-2351, Sep 1963 AD422 081.

[33] WAIT, J. R. Current-Carrying Wire Loops in a Simple Inhomogeneous Region. *Journal of Applied Physics* (Correspondence), vol 23, no 4, Apr 1952, pp 497–498.

[34] WAIT, J. R. Mutual Coupling of Loops Lying on the Ground. *Geophysics,* vol 19, no 2, Apr 1954, pp 290–296.

[35] WAIT, J. R. Induction by an Oscillating Magnetic Dipole over a Two-Layer Ground. *Applied Scientific Research,* vol B8, 1960, pp 213–253.

[36] WAIT, J. R. Mutual Electromagnetic Coupling of Loops over a Homogeneous Ground. *Geophysics,* vol 20, no 3, Jul 1955, pp 630–637.

[37] HOHMANN, G. W. Inductive EM Methods in *Electromagnetic Exploration of the Moon,* Edited by W. I. Linlor. Baltimore: Mono Book Corp, 1970, pp 171 – 190. See also *Geophysics,* Special Issue on EM Scattering, Feb 1971.

[38] WILLIAMS, R. H., and BENNING, C. J. Conductivity Measurements of the Earth at ELF. *IEEE Transactions on Antennas and Propagation,* vol AP-11, no 3, May 1963, pp 364–365.

[39] WAIT, J. R. A Conducting Sphere in a Time Varying Magnetic Field. *Geophysics,* vol 16, Oct 1951, pp 666–672.

[40] WAIT, J. R. *Method of Geophysical Exploration.* US Patent no 2,735,980 filed Dec 1953.

[41] VANYAN, L. L. *Electromagnetic Depth Soundings* (translation from Russian by G. V. Keller) New York: Plenum Press, 1969.

[42] *Electromagnetic Probing in Geophysics,*

J. R. Wait editor. Boulder, Colo.: Golem Press, 1971

[43] KING, R. W. P. *Electromagnetic Engineering Vol. I Fundamentals.* New York: McGraw-Hill, 1945.

[44] KIRKSCETHER, E. J. Ground Constant Measurements Using a Section of Balanced Two-Wire Transmission Line. *IRE Transactions on Antennas and Propagation,* vol AP-8, May 1960, pp 307 – 312.

[45] IIZUKA, K., and KING, R. W. P. An Experimental Study of the Properties of Antennas Immersed in Conducting Media. Scientific Rprt no 2, Cruft Lab, Harvard University, Cambridge, Mass., under contract AF 19(604)7262, Dec 16, 1961, AD273 687.

[46] VON HIPPEL, A. *Dielectric Materials and Applications.* Cambridge, Mass.: MIT Technology Press, 1954; New York: Wiley, 1954.

[47] KING, R. W. P. *Transmission-Line Theory.* New York: Dover, 1955.

[48] TAYLOR, J., HAN, C. C., TIEN, C. L., and HAGN, G. Open-Wire Transmission Lines Applied to the Measurement of the Macroscopic Electrical Properties of a Forest Region. Special Technical Rprt 42, TR-ECOM-0220-42, Stanford Research Institute, Menlo Park, Calif., Oct 1971, Reports control Symbol OSP-1366.

[49] BANNISTER, P. R. Surface-to-Surface and Subsurface-to-Air Propagation, Quasi-Static and Near Field Region. AGARD Conference Proceedings, no 20, *Subsurface Communications,* Paris, (Apr 1966) AD669 768, pp 309 – 338.

[50] WAIT, J. R. Electromagnetic Fields of a Dipole over an Anisotropic Half-Space. *Canadian Journal of Physics,* vol 44, 1966, p 2387.

[51] GALEJS, J. Horizontally Oriented Antennas in the Presence of an Anisotropic Ground. *Radio Science,* vol 4, Nov 1969, pp 1047 – 1059.

[52] deBETTENCOURT, J. T., and FRAZIER, J. W. Rock Electrical Characteristics Deduced from Depth Attenuation Rates (in Drill Holes). *IEEE Transactions on Antennas and Propagation,* vol AP-11, no 3, May 1963, pp 358 – 363.

[53] SEGESMEN, F., SOLOWAY, S., and WATSON, M. Well-Logging — The Exploration of Subsurface Geology. *Proceedings of the IRE,* vol 50, no 11, Nov 1962, pp 2227 – 2243.

[54] WAIT, J. R. The Magnetic Dipole Antenna Immersed in a Conducting Medium. *Proceedings of the IRE,* vol 40, Oct 1952, pp 1244 – 1245.

[55] WAIT, J. R. Insulated Loop Antenna Immersed in a Conducting Medium. *NBS Journal of Research,* vol 59, no 2, Aug 1957, pp 133 – 137; and vol 68D, no 11, Nov 1964, pp 1243 – 1250.

[56] MOORE, R. K. The Theory of Radio Communication Between Submerged Submarines. Ph.D. thesis. Cornell University, Ithaca, New York, 1951.

[57] TSAO, C. K. H., and deBETTENCOURT, J.T. Impedance of a Finite Length Insulated Dipole in Dissipative Media. Conference Proceedings on *Environmental Effects on Antenna Performance,* vol I, Boulder, Colo., Jul 1969, pp 72 – 76.

[58] TSAO, C. K. H., and deBETTENCOURT, J.T. Measurement of Conductivity With Electrically Short Probes. *IEEE Transactions on Instrumentation and Measurement,* vol IM-16, no 3, Sep 1967.

[59] KING, R. W. P. Dipoles in Dissipative Media. Cruft Lab, Harvard University, Cambridge, Mass., Tech Rprt no 336, Feb 1, 1961.

[60] KING, R. W. P., HARRISON, C. W., and DENTON, D. H. The Electrically Short Antenna as a Probe for Measuring Free Electron Densities and Collision Frequencies in an Ionized Region. *NBS Journal of Research,* vol 65D, Jul/Aug 1961, pp 371 – 384.

[61] TSAO, C. K. H., and deBETTENCOURT, J.T. Measurement of the Phase Constant for Rock-Propagated Radio Signals. *IEEE Transactions on Communication Technology,* vol COM-15, no 4, Aug 1967, pp 592 – 597.

[62] WAIT, J. R., and FULLER, J. A. On Radio Propagation Through Earth. *IEEE Transactions on Antennas and Propagation,* vol AP-19, no 6, Nov 1971, pp 796 – 798.

[63] GRUBB, R. N., and WAIT, J. R. In Situ Measurements of the Complex Propagation Constant in Rocks for Frequencies from 1 to 10 MHz. *Electronic Letters,* vol 7, no 17, 26 Aug 1971, pp 506 – 507.

[64] TSAO, C. K. H., and deBETTENCOURT, J.T. Subsurface Radio Propagation Experiments. AGARD Conference Proceedings, no 20, *Subsurface Communications.* Paris, Apr 1966, AD 669 768, pp 471 – 480.

Also, *Radio Science,* vol 3, no 11, Nov 1968, pp 1039 – 1044.

[65] KING, R. W. P. *The Theory of Linear Antennas.* Cambridge, Mass.: Harvard University Press, 1956, chap VII.

[66] WAIT, J. R. Mutual Electromagnetic Coupling of Loops Over a Homogeneous Ground. *Geophysics,* vol 21, no 2, Apr 1956, pp 479 – 484.

[67] SPIES, K. P., and WAIT, J. R. Determining Electrical Ground Constants from the Mutual Impedance of Small Co-Planar Loops, *IEEE Transactions on Antennas and Propagation,* vol AP-20, Jul 1972, pp 501 – 502.

[68] GOLDSTEIN, N. E., PARKER, H. W., and HAGN, G. H. Three Techniques for Measurement of Ground Constants in the Presence of Vegetation. Special Technical Rprt 30, Stanford Research Institute, Menlo Park, Calif., Contract DA-36-039, AMC-00040 (E), Mar 1967, AD672 496.

Appendix A
Mutual Impedance of Loops on the Ground

(These Appendixes are not part of IEEE Std 356-1974, Guide for Radio Methods of Measuring Earth Conductivity.)

A.1 Purpose and Scope. We give here the expressions for magnetic and electric fields at points on the plane interface between two semi-infinite homogeneous media which points are distant from a small horizontal loop (vmd)[7] source situated on the interface. The underlying theory is the early pioneering work of Arnold Sommerfeld.

The expressions then yield the mutual impedance between horizontal coplanar loops or between a loop and a horizontal wire broadside to the direction of propagation. The mutual impedance then may be used to deduce the electrical constants of the ground.

A.2 Theory. Wait, Ref [33], developed closed form and exact expressions for the electromagnetic fields from an isolated loop located on the boundary (plane) of two semi-infinite homogeneous media. He acknowledged that his development was based on a special evaluation of the integral formulation due to Sommerfeld. (See also King, Ref [65].) We present the results when the upper medium (medium 1) is lossless air and the lower medium (medium 2) is conducting ground.

It is customary to assume that the media are nonmagnetic so that the permeabilities μ_i are the same as that for free space $\mu_o = 4\pi \times 10^{-7}$ H/m. For an implied time variation $e^{j\omega t}$, the complex propagation constants are given by

$$\gamma_i^2 = -\beta_o^2 \epsilon'_{r_i} \qquad i = 1, 2 \qquad \text{(Eq A1)}$$

where

β_o = the real phase constant of free space
$= \omega(\mu_o \epsilon_o)^{1/2} = 2\pi/\lambda_o$

$\epsilon_o = 10^{-9}/36\pi$ F/m

$\omega = 2\pi f$

f = the wave frequency

λ_o = the free space wavelength
$= v_o/f$

v_o = the velocity of light in vacuo
$= 3 \times 10^8$ m/s

In Eq A1, $\epsilon_{r\,i}'$ is the complex relative dielectric constant (for medium i relative to that for free space). It is written as

$$\epsilon_{r\,i}' = \epsilon_{r_i}(1 - j\,p_i) \qquad \text{(Eq A2)}$$

where

ϵ_{r_i} = the real relative dielectric constant of medium i

p_i = the loss tangent of medium i

$$= \sigma_i/\omega\epsilon_o\epsilon_{r_i} = 1.8 \times 10^{10}\sigma_i/\epsilon_{ri}f$$

$$= 60\,\sigma_i\,\lambda_o/\epsilon_{r_i} \qquad \text{(Eq A3)}$$

where σ_i is the conductivity of medium i. It is customary to assume that for air $\sigma_1 = p_1 = 0$. Hence, from Eq A1, $\gamma_1{}^2 = -\beta_0^2$; from this we deduce $(\gamma_2/\gamma_1) = (\epsilon_{r\,2}')^{1/2} = n_2$ where n_2 is known as the complex refractive index of medium 2 relative to that of medium 1.

Let the source of fields be an electrically small loop of cross-sectional area A_T with N_T turns, and carrying a uniform current I_T. At points on the boundary $(\rho, \varphi, 0)$, Wait, Ref [66], shows that the vertical magnetic field strength is

$$H_z = \frac{I_T N_T A_T}{2\pi(\gamma_1^2 - \gamma_2^2)\,\rho^5}\,[\Phi_H(x_1) - \Phi_H(x_2)] \qquad \text{(Eq A4)}$$

where

$x_i = \gamma_i \rho$

$\Phi_H(x) = (9 + 9x + 4x^2 + x^3)e^{-x}$

Similarly, the only component of electric field is the tangential component given by

$$E_\varphi = j\omega\mu_o \frac{I_T N_T A_T}{2\pi\,(\gamma_1^2 - \gamma_2^2)\,\rho^4} \times [\Phi_E\,(x_1) - \Phi_E\,(x_2)] \qquad \text{(Eq A5)}$$

where

$\Phi_E(x) = (3 + 3x + x^2)e^{-x}$

At the point $(\rho, \varphi, 0)$ we may place a small horizontal loop, responsive to H_z, or a small horizontal wire perpendicular to the radius vector and responsive to E_φ. Considering the first case, let the receiving loop be electrically small having N_R turns and cross-sectional area

[7] The abbreviation vmd signifies vertical magnetic dipole. See Footnote 5.

A_R. The open-circuit induced voltage V_R is

$$V_R = -j\omega\mu_o N_R A_R H_z \qquad \text{(Eq A6)}$$

with H_z given by Eq A4. The mutual impedance between the coplanar transmitting and receiving loops then may be written as

$$Z_m = \frac{V_r}{I_t} = j\omega\mu_o \frac{N^2 A^2}{2\pi(\gamma_1^2 - \gamma_2^2)\,\rho^5} \times [\Phi_H(x_2) - \Phi(x_1)] \qquad \text{(Eq A7)}$$

where $N = N_T = N_R$ and $A = A_T = A_R$. (For an electrically short horizontal wire, expressions similar to Eqs A6 and A7 may be readily developed for the between coplanar loop and wire.)

Most field measurements of earth conductivity are made at frequencies below 30 MHz. At such frequencies the noise fields incident upon a small loop (or wire) receiving antenna near the surface will be polarized with the incident electric vector in the vertical plane and practically perpendicular to the earth's surface, and with the magnetic field vector tangential to that surface. Hence the fields radiated from a small transmitting loop will be favorably oriented for optimum signal-to-noise ratio at the receiving antenna of the types suggested. This is an advantage for techniques employing horizontal transmitting loops near the ground.

For sufficiently low frequencies at which the ground is highly conducting, expressions were developed for obtaining the earth conductivity from measurements of the mutual impedance of coplanar loops (Ref [4], chapter V). The resulting equations were simplifications of Eq A7, and will be given following a discussion of the more general case developed more recently by Spies and Wait [67]. In their treatment of coplanar loops lying on the plane air—ground interface, the mutual impedance given by Eq A7 was normalized to that value Z_o for the mutual impedance between coplanar loops in free space. For identical lossless loops, Z_o is a pure reactance given by

$$Z_o = j\omega M \qquad \text{(Eq A8)}$$

where M is the mutual inductance

$$M = \mu_o N^2 A^2/(2\pi\rho^3) \qquad \text{(Eq A9)}$$

Substituting Eq A8, with Eq A9, into Eq A7, the normalized impedance ratio (Z_m/Z_o) may be written

$$\frac{Z_m}{Z_o} = \frac{2}{(\gamma_1^2 - \gamma_2^2)\,\rho^2}\,[\varphi_H(x_2) - \varphi_H(x_1)] \qquad \text{(Eq A10)}$$

This ratio is generally complex and to illustrate its behavior curves of Z_m/Z_o for a given value of ρ/λ_o are plotted on a complex plane. An example for $\rho/\lambda_o = 0.1$, is shown in Fig A1.[8] Two sets of isocontours are shown; one for constant relative dielectric constant ϵ_r and the other set for a given ratio of σ/f. (The subscript 2 denoting the ground is dropped hereafter.)

If the real and imaginary parts of the mutual impedance Z_m can be measured independently, a plot of the normalized ratio on curve sheets such as Fig A1 will yield values of both ϵ_r and σ. In general, synchronization of received and transmitted signals will be required to determine relative phase; one method which may be employed is that used previously in the measurements of the phase of subsurface propagated signals (Ref [61]).

A.3 Approximate Relations for Large Loss Tangents. When the frequencies employed are sufficiently low, the loss tangets (p) of most earth media are large and simplified relations result; the electrical constant deduced is then the conductivity σ. Denoting this case by the relation $p \geqq 9$, then upon using Eq A3, this may be restated as

$$\sigma/f \geqq C_1\,\epsilon_r \qquad \text{(Condition I) (Eq A11)}$$

where $C_1 = 5 \times 10^{-10}$. Using typical soil constants, the maximum frequencies when $p = 9$ are 2×10^6, 2.5×10^5, and 4×10^4 for moist soil ($\epsilon_r = 10$, $\sigma = 0.01$), fresh water ($\epsilon_r = 80$, $\sigma = 0.01$), and dry soil ($\epsilon_r = 5$, $\sigma = 0.0001$), respectively.

For media with large loss tangents, it is preferable to rewrite Eq A1 as

$$\gamma^2 = j\omega\mu_o\sigma' \qquad \text{(Eq A12)}$$

where σ' is the complex conductivity given by

$$\sigma' = \sigma\,[1 + j\,(1/p)] \qquad \text{(Eq A13)}$$

[8]Similar curve plots for other ratios of ρ/λ_o and additional numerical data are available from Spies and Wait (National Oceanic and Atmospheric Administration, Environmental Research Laboratories, Boulder, Colorado, 80302).

If $p \geq 9$ (Eq A11), then $\sigma' \approx \sigma$ whence

$$\gamma \approx (1 + j)/\delta \qquad \text{(Eq A14)}$$

where δ is known as the metallic skin depth of the earth. It is given by

$$\delta = (2/\omega\mu_o\sigma)^{1/2} \qquad \text{(Eq A15)}$$

whence

$$\delta = C_2/(f\sigma)^{1/2} \qquad \text{(Eq A16)}$$

where $C_2 = 503$.

In evaluation of measurements of Z_m, the value of $\beta_o\rho$ must be small in order to obtain unambiguous results. Further, for simpler measurement systems the sensitivity (that is, $|V_R/I_T|$) is limited so that $\rho/\lambda_o << 1$. We may denote this condition by $\rho/\lambda_o \leq 0.1$, or

$$\rho \leq C_3/f \qquad \text{(Condition II) (Eq A17)}$$

where $C_3 = 3 \times 10^7$.

The resulting expression for Z_m, assuming Conditions I and II, depends upon the dimensionless quantity $|\gamma\rho|$, that is, upon the ratio ρ/σ.

A.3.1 $|\gamma\rho| << 1$. For the case of a large loss tangent for the earth (Condition I, Eq A11, which leads to approximation Eq A14 for γ), this condition $|\gamma\rho| << 1$ means that $\sqrt{2}\ \rho/\delta << 1$, where δ is given by Eq A16. This case may be denoted by $p/\delta \leq 0.4$, whence

$$\rho \leq C_4/(f\sigma)^{1/2} \qquad \text{(Condition III) (Eq A18)}$$

where $C_4 = 201$.

If $\rho << \delta$, as for the case here, then the relation $\rho << \lambda_o$ (Eq A17, Condition II) is implied. This may be seen from forming the ratio $|\gamma_o\rho/\gamma\rho| = |\gamma_o/\gamma| = |1/\epsilon_r'|$, or $\delta/\lambda_o = (\pi|\epsilon_r'|\sqrt{2})^{-2} << 1$. When $|\gamma_o\rho| << 1$ and $|\gamma_2\rho| << 1$, then convergent series expansions of the exponentials in $\Phi_H(x_2)$ may be employed, whence Eq A7 may be approximated (Ref [4], chapter V) by

$$Z_m = Z_o(1 + x_2^2/4 \cdots) \qquad \text{(Eq A19)}$$

upon employing terms in x_i up to x_i^4 plus the previous assumption that $|\epsilon_r'| >> 1$. The second term in the parenthesis may be neglected in this case, since $|x_2| = |\gamma\rho| << 1$. Hence Eq A 19 may be approximated by

$$Z_m/Z_o = 1 \qquad \text{(Eq A20)}$$

so that Z_m is a pure reactance. Z_m is the same as that value for free space, Z_o; thus Z_m is unaffected by the ground, and no information about earth constants can be deducted for this case. The measurement of Z_m at such short distances will yield the value of Z_o to be used for obtaining the normalized ratio Z_m/Z_o for other cases.

Using a complex plane sheet similar to Fig A1, for the given ratio ρ/λ_o, the measured values of the normalized impedance should fall near the point $(1 + j0)$.

As an example, for $p = 9$ and $\rho/\lambda_o = 0.01$, and with $\epsilon_r = 10$ so that $\sigma/f = 5 \times 10^{-9}$, precise calculations give $|\gamma\rho| = 0.598$ and $Z_m/Z_o = 1.02\ (1 + j\ 0.05)$. Approximation A20 is close to this more precise value of Z_m/Z_o.

A.3.2 $|\gamma\rho| >> 1$. This case may be denoted by $|\gamma\rho| > 9$, for example. With the assumption of large loss tangent in Eq A11 and with Eq A16, this relation may be written as $\rho/\delta \geq 2\pi$, that is,

$$\rho \geq C_5\ /\ (f_\sigma)^{1/2} \qquad \text{(Condition IV) (Eq A21)}$$

where $C_5 = 3160$.

Thus for this case ρ is smaller than λ_o but larger than δ. In this range the exponential in $\Phi_H(\gamma\rho)$ cannot be expanded readily in a series; but for large loss tangent (p), this functional term in Eq A7 may be neglected because of the exponential damping $\exp(-\rho/\delta)$. It may then be shown (Ref [4] chapter V), that Eq A7 becomes

$$Z_m = \frac{(3NA)^2}{2\pi\sigma\rho^5} \qquad \text{(Eq A22)}$$

that is, Z_m is resistive, varying inversely as $(\sigma\rho^5)$. Hence from Eqs A22 and A8 with Eq A16, the normalized mutual impedance is given approximately by

$$\frac{Z_m}{Z_o} = -j9\ (\rho/\delta)^{-2} \qquad \text{(Eq A23)}$$

For this case, a plot of measured normalized impedance points would lie along the negative imaginary axis of a curve sheet similar to Fig A1, for the appropriate ratio ρ/λ_o.

A sample comparison of approximate and precise values of Z_m/Z_o was made for $\rho/\lambda_o = 0.1$ and loss tangent $p = 36$. For $\epsilon_r = 10$ so that $\sigma/f = 2 \times 10^{-8}$, then $|\gamma\rho| = 11.924$ and the precise value of $Z_m/Z_o = -j\ 0.122\ (1-j0.016)$. For $\rho = 3 \times 10^7/f$ and the aforementioned

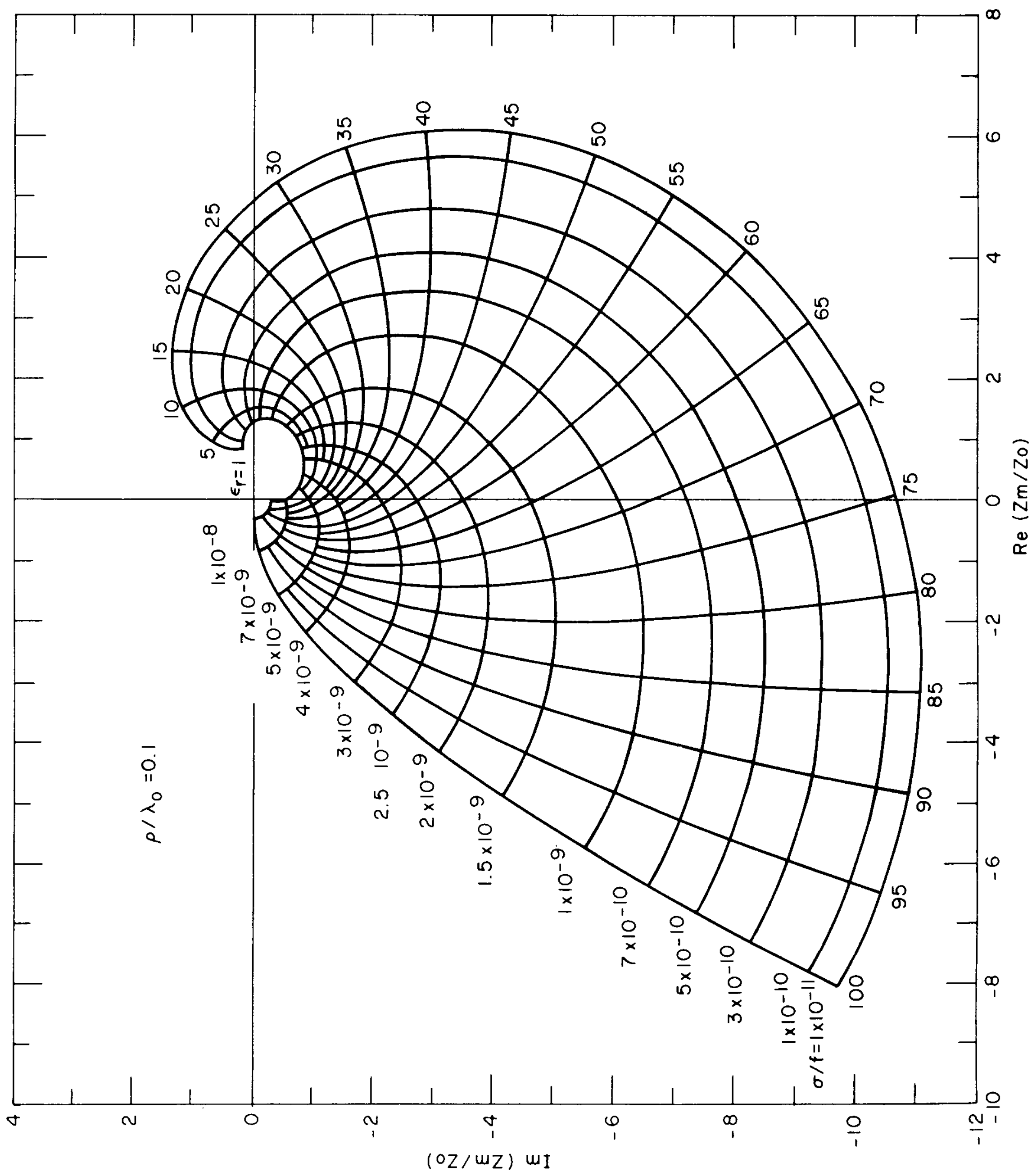


Fig A1
Complex Z_m/Z_o Plot for $\rho/\lambda o = 0.1$ (from Spies and Wait, Ref [67])

ratio for σ/f, the ratio $\rho/\delta = 8.43$; the approximate normalized impedance, from Eq A23, is $Z_m/Z_o = -j0.127$, in good agreement with the more precise value.

A.4 A Measurement Scheme. In field measurements which employ Eq A22 for data analysis, the maximum value of σ depends upon the system sensitivity or minimum measurable value of Z_m. Let R_m denote this sensitivity, that is, $R_m = |V_{R_o}/I_T|$ where V_{R_o} is the minimum detectable open-circuit antenna voltage at the receiver for a current I_T in the transmitting loop. As an example V_{R_o} is typically 1 μV; in measurements where I_T = 1 A, then $R_m = 10^{-6}$ Ω. For identical 3 m diameter loops and with the number of turns set at $N = 500$, then from Eq A22, the measured value of σ would be

$$\sigma = C_6/(R_m{}'\rho^5) \qquad \text{(Eq A24)}$$

where

$C_6 = 1.79 \times 10^7$

$R_m{}' = |V_R/I_T|$, V_R being the measured open-circuit received antenna voltage ($R_m' \geq R_m$).

If $R_m{}' = R_m = 10^{-6}$ Ω, then the largest measurable value of σ is

$$\sigma = C_7/\rho^5 \qquad \text{(Eq A25)}$$

where $C_7 = 1.79 \times 10^{13}$ for the loop geometry assumed.

Hence the range of σ obtainable depends upon ρ which in turn must exceed the skin-depth δ (Condition IV, Eq A21) and also must be less than the free-space wavelength λ_o (Condition II, Eq A17). The maximum permissible distance ρ_M, from Eq A17 is C_3/f; if $\rho = \rho_M$, the minimum discernible value of σ denoted by σ_m is

$$\sigma_m = C_8 f^5 \qquad \text{(Eq A26)}$$

where $C_8 = 7.36 \times 10^{-25}$.

The minimum permissible value of ρ, denoted by ρ_m is $C_5/(f\sigma)^{1/2}$, from Eq A21; if $\rho = \rho_m$, the maximum discernible value of σ is obtained by solving for σ from the relation $\sigma = C_7/\rho_m{}^5$. Denoting this value by σ_M, then

$$\sigma_M = C_9 f^{-(5/3)} \qquad \text{(Eq A27)}$$

where $C_9 = 678$.

Accordingly, the range of measurable σ, when $R_m' = R_m = 10^{-6}$ Ω for the assumed loop geometries, may be written as

$$\sigma_m \leq \sigma \leq \sigma_M \qquad \text{(Eq A28)}$$

where σ_m is given by Eq A26 and σ_M by Eq A27, subject to Conditions I (large loss tangent), II ($\rho/\lambda_o \leq 0.1$), and IV ($\rho/\sigma \geq 2\pi$) as given by Eqs A11, A17, and A21, respectively,

The range of σ for these conditions can be extended by decreasing R_m and increasing the product ($N \times A$) for each loop. Decreasing R_m can be accomplished by decreasing the minimum detectable open-circuit loop antenna voltage V_{R_o} at the receiver, or increasing the transmitting loop antenna current I_T, or both.

The method can be used for measuring the apparent conductivity σ_a of a horizontally stratified ground where σ above becomes σ_a. Techniques for obtaining overburden and rock conductivities for a two-layer or three-layer model were discussed in Section 2 of the main text.

A.5 Further Discussion. The employment of loops has the advantage over horizontal wire dipole usage in portability and freedom from struggles encountered when stringing out horizontal wires through brambles, brush, ponds, and the like. In mutual impedance methods loop impedances are relatively less affected by variations in ground electrode resistance than are the impedances of horizontal wire dipoles. Both loops lying in the horizontal plane and horizontal electric dipoles will be less affected by atmospheric noise than when such antennas are oriented in the vertical plane because atmospheric noise is predominantly vertically polarized. Distance limits are imposed by signal-to-noise ratios at the operating frequencies. Actual sizes and number of turns of loops are also dictated by circuit matching considerations of impedance as a load on the transmitter and of the impedance of the receiver antenna versus its detector input impedance.

Appendix B
Transmission-Line Methods

B.1 Purpose and Scope. The input impedance of a transmission line is greatly affected by the electrical properties of the medium between the wires or cylinders comprising the line. We discuss that technique which employs closely spaced parallel wires for the transmission line. In particular, we discuss that method which employs the measurements of input impedances when the line is unterminated and when it is terminated in an ideal zero impedance short circuit. Data from both sets of measurements then may be used to deduce the conductivity, dielectric constant, and loss tangent of the medium.

Kirkscether, Ref [44], outlined several methods which employed parallel-wire transmission lines inserted into the earth. Results with short sections of unterminated lines were emphasized. He presented exact and approximate procedures and discussed their advantages and limitations. Among the latter was the need for exact values of inserted line length s in the ground.

We discuss now the method where the line length need not be known in order to deduce σ and ϵ_r of media such as swamps, lakes, water-filled drill holes, and forest vegetation. The use of the method is illustrated by measurements employing a commercially available balanced 300 Ω air-dielectric parallel-wire line with porcelain spreaders, inserted into a water-filled drill hole (Ref [4], chapter V). The method has been used to measure the electrical properties of tropical forestation, along with other methods discussed in our text, by Hagn and co-workers (Refs [68], [48]).

B.2 Theory. The method and analysis is based on extension of relations given in several texts and we use those given in King, Ref [46], chapter II. The characteristic impedance Z_c and current propagation constant γ for a uniform line are

$$Z_C = (z/y)^{1/2} = |Z_C| \exp(j\varphi_c) \qquad \text{(Eq B1)}$$

$$\gamma = (zy)^{1/2} = \alpha + j\beta \qquad \text{(Eq B2)}$$

where

z = series impedance per unit length of line
$= r + jx = r + j\omega l = Z_C \gamma$ (Eq B3)

y = shunt admittance per unit length of line
$= g + jb = g + j\omega c = \gamma / Z_C$

with

r = series resistance per unit length of line
l = series inductance per unit length of line
g = shunt conductance per unit length of line
c = shunt capacitance per unit length of line

In Eq B2 α and β are the attenuation and the phase constant, respectively. For good conductors, γ is the same as the wave propagation constant of the ambient medium.

Let Z_{OC} be the input impedance when the line is unterminated (open-circuit termination) and Z_{SC} that when the line is terminated by zero impedance (short-circuit termination). Then

$$Z_C = (Z_{SC}\, Z_{OC})^{1/2} \qquad \text{(Eq B4)}$$

$$\tanh(\gamma s) = (Z_{SC}/Z_{OC})^{1/2} \qquad \text{(Eq B5)}$$

Let us form the ratios $\eta = r/\omega l$ and $p = g/\omega c$, where η is the reciprocal of the series Q_L of the transmission line. For conductors imbedded in a lossy medium [Refs [4], [47]), one has $g/\sigma = c/\epsilon_o \epsilon_r$, so that p becomes

$$p = \sigma / \omega\, \epsilon_o \epsilon_r \qquad \text{(Eq B6)}$$

that is, the ratio p defined for the embedded line is the same as the loss tangent of the dissipative medium. If the experimental arrangement yields the value of p and ϵ_r, then σ is obtained via Eq B6.

Forming the square of Eq B1 and using the line constants from Eq B2 with the definitions of η and p, one obtains

$$\left.\begin{aligned} |Z_C|^2 &= (l/c)\,[(1+\eta^2)/(1+p^2)]^{1/2} \\ 2\varphi_c &= \tan^{-1}[(p-\eta)/(1+\eta p)] \end{aligned}\right\} \qquad \text{(Eq B7)}$$

An experimental value of Z_C is obtained from measured values of Z_{SC} and Z_{OC}, via Eq B4. (A knowledge of line length s will yield an experimental value of γ from the same measurements, via Eq B5).

For measurements of the line in air, where the loss tangent is zero, the characteristic impedance is denoted by $Z_o = |Z_o| \exp(j\varphi_o)$. Letting the subscript o denote values for the line in air, the loss tangent $p_o = 0$ and $\eta_o = r_o/\omega l_o$.

From Eq B7 then, $|Z_o|^2 = (l_o/c_o)(1 + \eta_o^2)^{1/2}$ and $\eta_o = -\tan(2\varphi_o)$.

B.3 Method. In essence, the experimental method (Ref [4], chapter V) consisted of comparing measurements of Z_C when the line was immersed in the dissipative medium with that value Z_o for the line in air. For high-quality lines, simplification results because $\eta_o^2 << 1$ from measured values of φ_o ($\varphi_o < 0$). Further, it is assumed that the medium does not change the value of z from its value z_o (a major effect would be to change l if the relative magnetic permeability μ_r greatly exceeded unity). The value of η for the line in the medium is therefore assumed to be its value in air, whence $\eta^2 << 1$.

The line was inserted into the well water, connected to an RG-22/U feeder whose values of Z_C and propagation constant were previously measured. The input impedance of the RG-22/U so connected was measured[9] when the parallel wire line was open circuited and then short circuited. From these impedances and the characteristics of the feedline, measured values of the open- and short-circuited impedances Z_{OC} and Z_{SC} at the input of the immersed test line can be calculated.

The data on Z_{SC} and Z_{OC}, inserted into Eq B4, gave values of Z_C for which $\varphi_c > 0$. This means that $p > \eta$. (It turned out that approximate values of p were such that $p >> \eta$ and $p\,\eta << 1$.) With these assumptions, Eq B7 becomes

$$|Z_c|^2 \approx (l_o/c)(1 + p^2)^{-1/2}$$
$$2\varphi_c \approx \tan^{-1}(p - \eta_o) \qquad \text{(Eq B8)}$$

Forming now the ratio $(Z_C/Z_o)^2 = |Z_C/Z_o|^2 \exp(j2\varphi_1)$ and noting $c = c_o \epsilon_r$ then

$$p \approx \tan(2\varphi_1) = \eta_o + \tan(2\varphi_c) \qquad \text{(Eq B9)}$$

and

$$\epsilon_r = |Z_C/Z_o|^{-2}\cos(2\varphi_1) \qquad \text{(Eq B10)}$$

Thus, having deduced p and ϵ_r from the measurements, σ is obtained from Eq B6. Often $\eta_o << p$ so that Eqs B9 and B10 may be simplified still further.

[9] Three-terminal measurements were required because the RG-22/U is a balanced cable whereas the rf bridge (General Radio Co 916A) employed was not. A wide band balun transformer of suitable design might well be employed.

B.4 Results and Discussion. Measured values of $|Z_C|$ and φ_c were plotted as a function of frequency and smoothed curves drawn through the points. From smoothed values of $|Z_C|$ and φ_c so obtained, ϵ_r, p, and σ were computed and are shown plotted on the curves of Fig B1.

The results and trends agree reasonably well with some water sample measurements made in the laboratory using a short coaxial condenser and a parallel-plate capacitor. (Measurements with the latter showed slightly lower values at lower frequencies due in part to polarization effects at the electrodes.)

Measurement accuracy is enhanced if $|Z_{OC}| \approx |Z_{SC}| = |Z_C|$, the design impedance level at which bridges are most accurate. But

$$Z_{OC} = Z_c \coth(\gamma s) \qquad \text{(Eq B11)}$$

$$Z_{SC} = Z_c \tanh(\gamma s) \qquad \text{(Eq B12)}$$

For small losses $\gamma \approx j\,2\pi/\lambda$, where λ is the wavelength in the medium. For optimum accuracy, $s \approx \lambda/8$. Accuracy is reduced if the measuring frequency extends over too wide a band about the optimum, for a given line length.

While the method is suitable for media such as lakes and forest vegetation (Ref [69]), it has less applicability when measuring compact earth soils. In this case, the simplified method of Kirkscether, Ref [44], with electrically short lines finds wider acceptability in the field. One measures Z_{OC} for a line inserted into the earth, having obtained its capacitance per meter c_o in air. If $|\gamma s| << 1$, Eq B11 is simplified. Forming the reciprocal of Z_{OC} yields $Y_{OC} = G_{OC} + jB_{OC}$. Then,

$$Y_{OC} \approx ys = (g + j\,\omega\,c)s \qquad \text{(Eq B13)}$$

from which σ and ϵ_r may be determined, namely:

$$\epsilon_r = (B_{OC}/\omega)(c_o s)^{-1} \qquad \text{(Eq B14)}$$

$$\sigma = (G_{OC}\,\epsilon_o)(c_o s)^{-1} \qquad \text{(Eq B15)}$$

The electrically short unterminated line behaves then as a lossy capacitor. The wires should be rigid, preferable of tubular construction, and the inserted line must be designed to maintain uniform and known spacing. The resulting values of σ and ϵ_r are those averaged for cylindrical volume approximately one line length long and radius two to three times the line separation. The results apply to near surface areas, of depth comparable to line length.

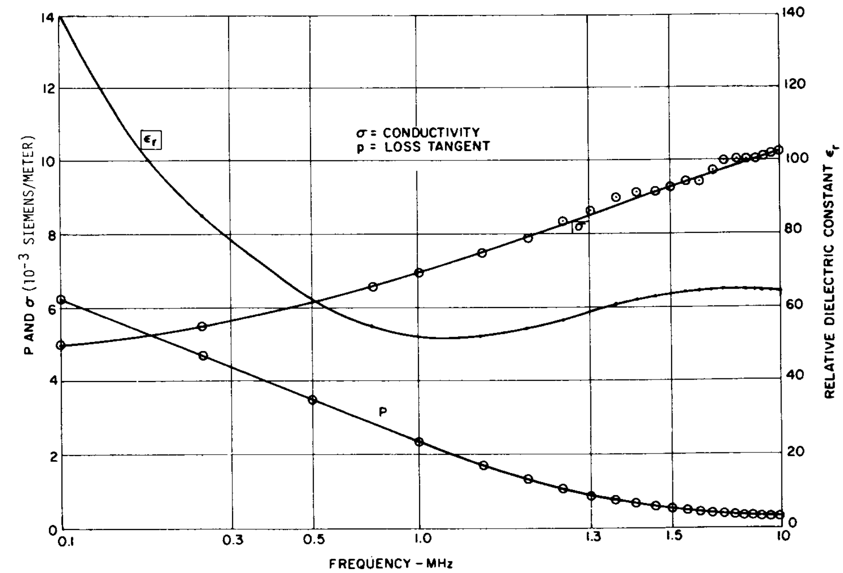


Fig B1
Electrical Characteristics of Drill Hole Water (Goffstown, N.H.), June 1961. Parallel-Wire Transmission Line Method (Ref [4])

Appendix C
List of Principal Letter Symbols and Special Terminology

C.1 List of Principal Letter Symbols

Symbol	*Definition*
A	An area of a loop. Appropriately subscripted
B	Susceptance, of a transmission line, defined in text
$C_1, C_2, \cdots C_9$	Various constants defined in the text
elf, lf, mf, hf, vhf, uhf	Letter designations for the extremely low, low, medium, high, very-high, and ultra-high frequency bands
$E_{h,v}$	Horizontal, vertical electric field strength
E_ϕ	ϕ - component of electric field strength
G	Conductance, of a transmission line, defined in text
H_h	Horizontal magnetic-field strength
$H_{x,y,z}$	Magnetic-field strength, cartesian components
I	A current in a wire. Appropriately subscripted
K	A reflection coefficient, defined for layered conducting media
M	Mutual inductance of a loop

N	Number of turns in a loop
Q	A ratio of magnetic-field components ($Q = H_x / H_z$)
R	A distance
R_m, R_m'	A mutual resistance, defined in the text
V	Voltage in a loop or wire, as subscripted
Y	Admittances, defined in text, for transmission lines.
Z, Z_m, Z_o ···	Impedances, defined in the text
b	Susceptance per unit length
c	Shunt capacitance per unit length
d	A distance or a layer thickness, as defined in the text
f	Radio wave frequency
g	Shunt conductance per unit length
j	$= \sqrt{-1}$
l	Series inductance per unit length
n	The (complex) refractive index of a medium (relative to air)
p	The loss tangent in a conductive medium ($p = \tan \Psi$)
r	Series resistance per unit length
s	A wire length
v_o	Velocity of light in vacuo $\approx 3 \times 10^8$ m/s
y	Admittance per unit length
z	Impedance per unit length
α'	Transverse anisotropy (of conductivity) ($\alpha' = \sigma_h / \sigma_v$)
α	Attenuation constant
β	Real phase constant ($\beta_o = 2\pi/\lambda_o$ in free space)
γ_i	Complex propagation constant for a medium designated by i. γ_i has dimensions $(\text{meter})^{-1}$.
δ	Skin depth
ϵ, ϵ_o	Permittivity (ϵ_o in vacuo). In air, $\epsilon \cong \epsilon_o \cong 10^{-9}/36\pi$ F/m
ϵ_r	Dielectric constant of a medium relative to air
ϵ'_r	Complex relative dielectric constant
ζ_a	An apparent complex surface impedance
η	A dissipation ratio
θ	A bearing angle
λ	Wavelength (λ_o = wavelength in free space)
μ_o	Permeability (μ_o *in vacuo*). In air, $\mu = \mu_o = 4\pi \times 10^{-7}$ H/m
ρ, ϕ, z	Cylindrical coordinates
σ	Conductivity (Siemens per meter) (variously subscripted)
$\overline{\sigma}$	Geometric mean conductivity
σ_a	Apparent conductivity
σ_h, σ_v	Conductivity in the horizontal, vertical direction
σ_x, σ_y, σ_z	Conductivities, cartesian components
Φ	A function of loop separation geometry
φ_c, φ_o	Argument of impedance
Ψ	Argument of the loss tangent ($\Psi = \tan^{-1} p$)
ω	Angular radian frequency (rad sec^{-1})

C.2 Special Terminology

magneto-telluric. An adjective denoting natural magnetic and electric fields, and effects produced by them. Abbreviated: M-T.

telluric. An adjective denoting the electric field effects in the earth due to M-T fields.

overburden. The surface layers or regions of the earth that are water bearing and are subject to weathering. They comprise predominantly sand, gravel, clays, and poorly consolidated rocks.

basement. The rock region underlying the overburden largely comprising aged rock types, often crystalline and of low conductivity.

Moho (Mohorovičić discontinuity). Seismic discontinuity situated about 35 km below the continents and about 10 km below the oceans. Crudely speaking, it separates the earth's crust and mantle.

IEEE Standards of Particular Interest to Antenna and Propagation Engineers

IEEE Std	Title
145-1973	Definitions of Terms for Antennas
146-1953	Definitions of Terms for Antennas and Waveguides
147-1955	Definitions of Terms for Waveguide Components
148-1959	Measurement of Waveguides and Components (Reaff 1971)
149-1965	Test Procedure for Antennas (Reaff 1971) (ANSI C16.11-1971)
187-1951	Open Field Measurement of Spurious Radiation from Frequency Modulation and Television Broadcast Receivers
211-1969	Definitions of Terms for Radio Wave Propagation
284-1968	Measuring Field Strength, Continuous Wave, Sinusoidal
285-1968	Measuring Phase Shift at Frequencies Above 1 GHz
287-1968	Precision Coaxial Connectors
291-1969	Measuring Field Strength in Radio Wave Propagation
299-1969	Recommended Practice for Measurement of Shielding Effectiveness of High-Performance Shielding Enclosures
302-1969	Methods for Measuring (Below 1000 MHz) Electromagnetic Field Strength
356-1974	Radio Methods of Measuring Earth Conductivity